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ACCELERATION OF BODIES IN COMBUSTIBLE MIXTURES

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An analytical solution is obtained for the problem of acceleration of a body in a closed tube filled with a detonating mixture of gases. It is assumed that the body enters the tube with a certain initial velocity sufficient for burning initiation in an annular space between the body and tube surfaces. The effect of the mixture parameters, the shape and mass of the body, and the integral dissipation of the total momentum and enthalpy of the flow on the finite values of the velocity and the acceleration length is analyzed.

1. In a number of recent studies [1, 2], it was proposed to use the principle of combustion, including detonation combustion, for acceleration of bodies. Detonation combustion has been assumed to occur in the annular space between the body surface and the internal surface of the guiding tube, which is completely filled with a detonating mixture of hydrogen- or hydrocarbon fuel-based gases. Experimental data on subsonic acceleration of bodies of moderate mass were presented in [1], and the results of numerical simulation of various regimes of supersonic and subsonic combustion were described in [2]. The present paper offers an analytical analysis aimed at determining the basic parameters which affect acceleration. In addition, the criteria of determination of the velocities and lengths of acceleration are obtained depending on the shape and mass of the accelerated bodies, the mixture parameters, etc.

We consider the principal flow pattern (Fig. 1). A body of mass m and mid-sectional area $S_{\rm b}$, which has the initial velocity V_0, enters a cylindrical tube of cross-sectional area S, which is filled with a combustible mixture. The mixture detonates behind a system of oblique shock waves between the tube wall and the body surface. As a result of energy production, the gas accelerates to velocity u and acquires a useful momentum. To determine u, it is necessary to solve a number of gas-dynamic problems in the regions located between cross sections 1–4, taking into account energy production in combustion region II.

The following parameters are assumed to be prescribed: the velocity of combustion products u, the density of the mixture ρ, the mid-sectional area of the body $S_{\rm b}$, the cross-sectional area of the tube S ($S \approx S_{\rm b}$), and the drag coefficient of the body C_x ($C_x \cong 0.1$).

The equation of motion for a body moving in the channel of an accelerator can be written as follows:

$$\frac{dV}{dt} = auV - (a+k)V^2. \tag{1.1}$$

Here $a = \rho S/m$, $k = C_x \rho S_{\rm b}/2m$ is the ballistic coefficient, m is the mass of the body (it was assumed in calculations that $a \cong 0.2$ m^{-1} for the pressure $p = 100$ atm, $S = 0.4$ m^2, and $m = 100$ kg).

The solution of Eq. (1.1) is

$$V = \frac{au}{a+k}\left\{1 - \left[1 - \frac{au}{(a+k)V_0}\right]\exp(-aut)\right\}^{-1}.$$

It follows from this equation that the drag of the body for small values of C_x and close values of S and $S_{\rm b}$ ($k \ll a$) has a small effect on the final velocity of acceleration, which depends strongly on the velocity of

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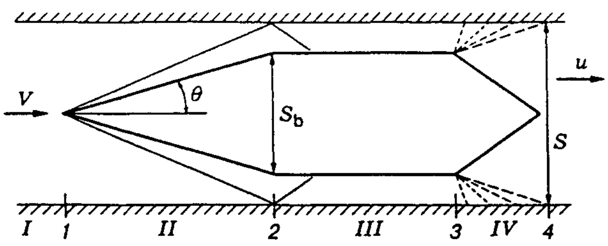

Fig. 1

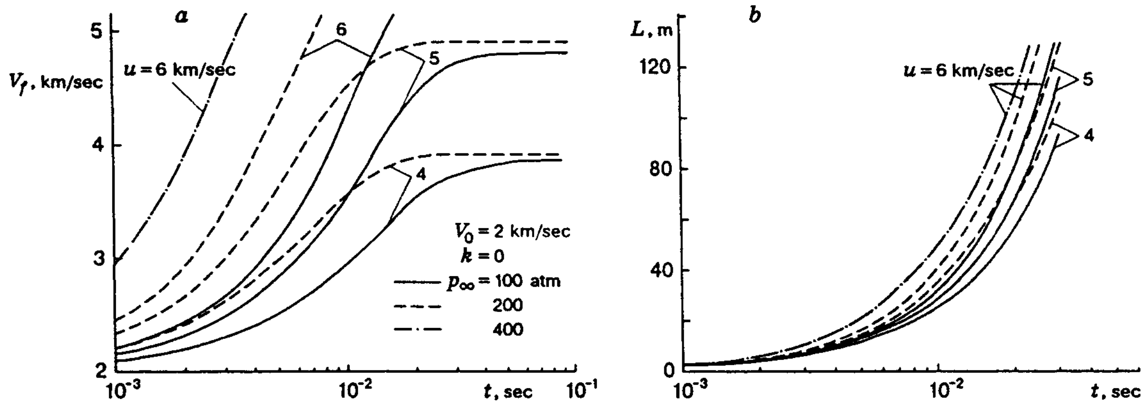

Fig. 2

combustion products u and the external pressure p (or the density ρ) (Fig. 2a). For the path L covered by the body, the solution is

$$L = \frac{au}{a+k}\left\{t + \frac{1}{au}\ln\frac{[1-(1-au/(a+k)V_0)\exp(-aut)]}{au/(a+k)V_0}\right\}. \tag{1.2}$$

It follows from (1.2) that the finite velocity of acceleration should be chosen based on a prior prescribed length of the tube L, which is limited by its weight and dimensions (Fig. 2b).

We rewrite Eq. (1.1) as follows:

$$\frac{dV}{dx} = au - (a+k)V.$$

Its solution can be represented in the following form:

$$\frac{au-(a+k)V_0}{au-(a+k)V} = \exp\left[(a+k)x\right]. \tag{1.3}$$

If $V = V_f$, where V_f is the finite velocity, i.e., the velocity of the accelerated body at the end of the tube, where $x = L \equiv x_p$ (x_p is the length of acceleration), it follows from solution (1.3) that, for given values of u, V_0, and V_f, there exists a family of bodies corresponding to a fixed value of the parameter $\rho S x_p/m$, from which we can determine either the length of acceleration from known ρ, S, and m or the mass m, which corresponds to a given value of x_p. The corresponding dependence is plotted in Fig. 3. The solution obtained extends the domain of similarity described in [1]: it is valid for all k, including $k = 0$.

Computations in [1] showed that, for a fixed acceleration of a body in a mixture of combustibles with prescribed concentrations, the ratio of the scale of various bodies and the size of the tube is the cubic root of the mass of the body. In particular, this follows from the form of the parameter $\rho S x_p/m$ for bodies whose shape is close to a sphere, i.e., when $m \sim AR^3$ and $S_b \sim BR^2$ (A and B = const). The length of acceleration x_p is inversely proportional to the density of the mixture ρ, and the mass of the mixture is directly proportional

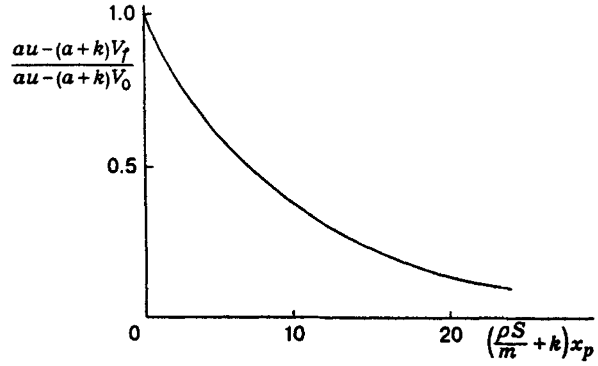

Fig. 3

to the mass of the body [1], which also follows from the relationship $\rho S x_p/m = \text{const}$.

However, more detailed information can be gained from the solution (1.3). We can perform a scale recalculation for arbitrary shapes and masses of bodies, taking into account the initial and finite velocities of acceleration, the real drag of the bodies, and the values of the parameters that characterize the mixture.

The drag of slender pointed bodies ($k \ll 1$) has hardly any effect on the values of the finite velocity of their acceleration if their shape does not change because of ablation.

2. In the more correct consideration, the flow velocity u behind the accelerated body should be determined from the system of integral balance of the mass, momentum, and energy fluxes, which takes into account the overall losses because of friction, heat transfer, and wave drag. The equation of motion of the body for a gas flow in cross sections 1 and 4 (see Fig. 1) takes the following form:

$$m\frac{dV}{dt} = T = p_4S_4(1+\gamma_4\mathrm{M}_4^2) - p_1S_1(1+\gamma_1\mathrm{M}_1^2). \tag{2.1}$$

Here p_i, S_i, γ_i, and M_i are the static pressure, the area, the ratio of the specific heats C_p/C_V, and the Mach number in cross sections $i = 1$, 2, 3, and 4, respectively ($S_1 = S_4$).

Similarly, by virtue of the continuity equation and the law of energy conservation written for the same cross sections, we obtain

$$\frac{p_1}{p_4}\frac{\mathrm{M}_1}{\mathrm{M}_4}\frac{S_1}{S_4}\sqrt{\frac{h_4}{h_1}\frac{(\gamma_4-1)}{(\gamma_1-1)}\frac{\gamma_1}{\gamma_4}} = 1; \tag{2.2}$$

$$\frac{h_4}{h_1} = \frac{(1-\zeta_q)(1+(\gamma_1-1)\mathrm{M}_1^2/2) + (q+\zeta_q h_w)h_1^{-1}}{1+(\gamma_4-1)\mathrm{M}_4^2/2}. \tag{2.3}$$

Here h_1 and h_4 are the static enthalpies of the flow of the combustible mixture in cross sections 1 and 4, h_w is the enthalpy on the body surface, q is the specific heat of fuel burning [3], $\zeta_q = Q_w/S_1\rho_1V_1(h_1 + V_1^2/2 - h_w)$, ζ_q is the dimensionless heat-transfer coefficient proportional to the Stanton number, Q_w is the integral heat flux to the body surface, and $\mathrm{M}_i^2 = V_i^2/(\gamma_i - 1)h_i$.

We introduce a dimensionless distance $\xi = x/l_p$, where l_p is the reference scale which has the physical meaning of the length on which the increment of the velocity of the body V_1 is equal to the local speed of sound, $l_p = m(\gamma_1 - 1)h_1/\rho_1S_1$. We pass to the variable ξ in Eq. (2.1) and then integrate it with allowance for the conservation laws (2.2) and (2.3).

We obtain

$$\frac{d\mathrm{M}_1^2}{d\xi} = (2\gamma_1\mathrm{M}_1^2)\left[\eta_4\sqrt{1-\zeta_q+\frac{2(1+\theta_1)}{(\gamma_1-1)\mathrm{M}_1^2}} - 1 - \frac{1}{\gamma_1\mathrm{M}_1^2}\right]. \tag{2.4}$$

Here $\theta_1 = [q - \zeta_q(h_1 - h_w)]h_1^{-1}$ and $\eta_4 = (1+\gamma_4^{-1}\mathrm{M}_4^{-2})[1+2(\gamma_4-1)^{-1}\mathrm{M}_4^{-2}]^{-1/2}$.

The coefficient η_4 depends little on the "tail" Mach number M_4 within the range of $1 \leqslant M_4 < \infty$. For example, for $\gamma_4 = 1.3$ the coefficient η_4 falls within the range $0.64 < \eta_4 < 1$ for all values of M_4 within the cited range and almost coincides with unity for $M_4 \gg 1$.

The condition $\eta_4 = \text{const}$ being satisfied, it follows from Eq. (2.4) that there is a universal dependence [to within the accuracy of $o(M_1^{-2})$] of the dimensionless length ξ on the Mach number M_4, the initial conditions, the coefficient of dissipation of the total energy, and the Mach number of the Chapman–Jouguet detonation which characterizes the combustible mixture:

$$(1+Z)\gamma_1\xi = (1-Z)^{-1}\left\{\ln\left[1-(1-Z)\frac{Y_0^2+1}{2}\right] - \ln\left[1-(1-Z)\frac{Y_1^2+1}{2}\right]\right\} + \ln\frac{Y_1}{Y_0}. \tag{2.5}$$

Here $Y_i = m_i + \sqrt{m_i^2+1}$, $m_i = Z_q M_i/M_\theta$, $Z = Z_q\eta_4^2$, $Z_q = 1-\zeta_q$, $M_\theta^2 = 2(1+\theta_1)(\gamma_1-1)^{-1}$, the subscript $i=0$ refers to the flow in the initial cross section of the tube, and the subscript $i=1$ refers to the flow in the cross section of the tube in front of the body which moves together with it.

We note that the solution (2.5) is valid for combustible mixtures of an arbitrary composition. It is of interest that dissipative and wave losses of the kinetic energy, which depend on several parameters (Stanton number, drag coefficient, etc.), are determined here by only one universal "coefficient of losses" Z.

The solution (2.5) can be written in functional form:

$$\frac{x}{l_p} = f\left(Z_q\frac{M_0}{M_\theta}; Z_q\frac{M_1}{M_\theta}; Z\right). \tag{2.6}$$

It follows from solution (2.6) that equal values of the reduced lengths ξ can be obtained in combustible mixtures of an arbitrary composition on condition that the values of the dimensionless parameters $Z_q M_0/M_\theta$, $Z_q M_1/M_\theta$, and Z coincide.

For small values of dissipative losses, i.e., for $Z \to 1$, relation (2.5) becomes substantially simpler:

$$2\gamma_1\xi = \frac{Y_1^2+Y_0^2}{2} + \ln\frac{Y_1}{Y_0}. \tag{2.7}$$

It follows from (2.5) and (2.7) that, for $\eta_4 \cong 1$ and $(1-Z) \ll 1$, a great number of the reference lengths l_p are needed to approach the limiting value $[M_1^2/M_\theta^2]_{\max}$, which is reached when the right-hand side of Eq. (2.4) vanishes:

$$\left[\frac{M_1^2}{M_\theta^2}\right]_{\max} = \frac{Z^2}{Z_q(1-Z)^2}. \tag{2.8}$$

In the case of large dissipative losses of the kinetic energy, i.e., for values of Z greatly different from unity, the resulting thrust force T decreases rapidly over several lengths l_p.

It follows from Eq. (2.8) that the maximum value of the relative Mach number of body acceleration M_1/M_θ depends greatly on the extent to which the coefficient of losses Z differs from unity. The values of Z, in turn, are primarily determined by heat-transfer losses (i.e., by the value of Z_q) since $\eta_4 \cong 1$.

We note that, for $T = 0$ and $\zeta_q = C_x = 0$, relations (2.1)–(2-3) completely coincide with the relations for the normal shock wave. Thus, the problem of determination of the limiting values $M_{1,\max}$ is similar to a study of the normal shock wave with heat release, and the problem of determination of the velocity of body acceleration in an arbitrary cross section of the channel x is similar to a study of the structure of the normal shock wave.

To find the limits of existence of a steady-state quasi-one-dimensional flow in a tube with an accelerated body, a complete system of the balance of mass, momentum, and energy fluxes in regions I–IV was analyzed. For the Mach number M_3, before the flow rotates on the tail part of the body, we obtain

$$\gamma_3 M_3^2 = \frac{\gamma_3 + \sqrt{\gamma_3^2 - (\gamma_3^2-1)(Z_q+m_1^2)C_\sigma^{-2}}}{1-\sqrt{\gamma_3^2-(\gamma_3^2-1)(Z_q+m_1^2)C_\sigma^{-2}}}. \tag{2.9}$$

Here the values of $C_\sigma = 1-(1-S_3/S_4)C_x/2 + (S_3/S_4)\gamma_1 M_1^2$ and Z_q are set without taking into account the losses because of heat transfer and friction in the tail section of region IV.

It follows from (2.9) that there is a characteristic value of the Mach number M_J that corresponds to the Chapman–Jouguet detonation. Below this value, a steady-state quasi-one-dimensional flow in a channel with an accelerated body is impossible. The value of the Mach number of thermal choking M_J is reached when the radicand in formula (2.9) vanishes:

$$M_J^2 = \frac{(\gamma_3^2 - 1)M_\theta^2}{Z_q + \gamma_3^2(C_\sigma^2 - Z_q)}. \quad (2.10)$$

Based on an analysis of the conservation laws on the "diffuser" forebody (in regions I and II) (see Fig. 1), we can obtain the minimum "critical" value of the Mach number $M_{2,\min}$ which ensures the starting of the accelerator diffuser. It is found from the relation

$$\gamma_1 M_{2,\min}^2 = \frac{2(1 + \gamma_1^{-1}) - S_2/S_1}{1 - (1 - S_2/S_1)C_x/2}. \quad (2.11)$$

Without analysis of the conservation laws, it is impossible to obtain relationships of the type (2.10) and (2.11). In particular, Knowlen and Hertzberg [1; 2, p. 175] faced this problem. They showed that the simplified "entropy-free" model "does not allow the calculation of the lower limit of the accelerator and the minimum Mach number M_1 at which the diffuser can be started."

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